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LIMIT CYCLES IN JAPANESE MACROECONOMIC DATA: POLICY IMPLICATIONS FROM THE VIEW OF BUSINESS CYCLES[1]

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ABSTRACT

This paper examines the existence of limit cycles in Japanese macroeconomic variables using a threshold autoregressive (TAR) model. Recent business cycle theories are grouped into two main categories: (1) real business cycle and (2) endogenous business cycle. Real business cycle theory, which is modelled by an autoregressive (AR) model, has a linear dynamic system with consecutive exogenous shocks that cause cyclical deviations from a growth trend. On the other hand, endogenous business cycle theory, which includes limit cycles characterized by a TAR model, is based on a nonlinear dynamic system that has a mechanism that induces complicated economic fluctuations endogenously. Which theory is appropriate has significant implications for policy makers. Accordingly, it is necessary to investigate whether economic fluctuations depend on an endogenous mechanism or on exogenous shocks. To investigate this, we test for linearity in Japanese macroeconomic variables. The linearity test in the paper distinguishes the TAR model from the AR model. We find that most fluctuations have TAR processes. The results indicate that most Japanese macroeconomic data have limit cycles, and imply that the government could control the business cycle in Japan, even though the cyclical behaviour is extremely complicated.

Key words: Business cycle theory, Limit cycles, TAR model

JEL Classification: C22, E32, E61

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LIMIT CYCLES IN JAPANESE MACROECONOMIC DATA: POLICY IMPLICATIONS FROM THE VIEW OF BUSINESS CYCLES

1. Introduction

This paper examines the existence of limit cycles in Japanese macroeconomic variables using a threshold autoregressive (TAR) model. Recent business cycle theories are grouped into two main categories: (1) real business cycle and (2) endogenous business cycle. In real business cycle theory, a stochastic business cycle model, which includes stochastic exogenous influences for causing cyclical deviations from a growth trend, and a linear dynamical system, are used to describe the actual complicated economic fluctuations. An autoregressive (AR) model, therefore, is utilized mostly to estimate a model based on this theory.

Endogenous business cycle theory, on the other hand, is based on a nonlinear dynamic system that has a mechanism that induces complicated economic fluctuations endogenously. Based on this theory, limit cycles characterized by a TAR model, or chaotic dynamical systems, are utilized to describe the actual complicated economic fluctuations. For the estimation of the chaotic dynamical systems, see Nishigaki, Ikeda, and Satake (2007). They find that the evidence for chaos in Japanese quarterly macro-economic time series is weak.

Which theory is appropriate has significant implications for policy makers. Accordingly, it is necessary to investigate whether economic fluctuations depend on an endogenous mechanism or on exogenous shocks. To investigate this, we test for linearity in Japanese macroeconomic variables. The linearity test in the paper distinguishes the TAR model from the AR model. We find that most of them have TAR processes, i.e. the two-regime or three-regime TAR processes. The results indicate that most Japanese macroeconomic data have limit cycles. This implies that the government could control the business cycle in Japan, even though its cyclical behaviour is extremely complicated, because the process follows not the AR model, but the limit cycle expressed by the TAR model.

The remainder of this paper is organized as follows. After a brief survey of recent business cycle theories, Section 2 presents the relationship between limit cycles and endogenous business cycle theory. Section 3 briefly explains the method of linearity testing. Section 4 uses an example of simulation to illustrate the TAR process. Section 5 reports our empirical results. Section 6 concludes the paper.

2. Limit Cycles and Business Cycle Theories

For decades, the aim of business cycle theory has been to model the basic underlying dynamics of economic fluctuations. The Samuelson-Hicks-type multiplier and accelerator mechanism explains the regular forces in oscillating time series, by using linear dynamic systems of difference equations, with appropriate assumptions in the economic structures. Although they succeeded in modelling the business cycles generated by the internal dynamics of the economy, they failed to describe all the fluctuations of actual business cycles, which were characterized by many more irregularities, not only in the context of the monotonicity of the cycle, but also in its amplitude and frequency. Accordingly, a stochastic exogenous influence had been thought to be included in the theoretically expressed linear systems.

The rational expectations theory of business cycles constitutes a stochastic business cycle model that describes the actual complicated cycles by including stochastic exogenous influences, in addition to the implicit regularity with the linear difference and differential systems. Typical examples of such models are Lucas (1975) and Prescott (1986).

However, in endogenous business cycle theory, many papers have utilized non-linear economic dynamics, such as chaotic dynamics or limit cycle theory, to model business cycles, and they have succeeded in explaining the seemingly irregular fluctuations in actual time series. The basic idea of utilizing limit cycles in business cycle theory was first demonstrated by Kaldor (1940), and mathematically formulated by Chang and Smith (1971) and Varian (1979). Since then, the theoretical interest of economists has expanded this literature, e.g. Lorentz (1993).

To see the basic dynamics of limit cycle theory, consider the following two dimensional ordinary differential equation systems.

$$\dot{x}_1 = f_1(x_1, x_2) \qquad (2.1),$$

$$\dot{x}_2 = f_2(x_1, x_2) \qquad (2.2),$$

or, in vector notation,

$$\dot{x} = f(x) \qquad (2.3)$$

where $x = (x_1, x_2)$ and $f_i : U \to R^2$ are smooth functions defined in $U \subseteq R^2$. The dot over a variable denotes a time-derivative.

A solution curve, trajectory, or orbit is defined $\phi(t, x^0)$: i.e. when a certain initial point x^0 is given, $\phi(t, x^0)$ provides the value of x at time t. For the existence and uniqueness of a solution curve, the function f is assumed to be C^2.

A solution curve is called a periodic solution of period *T*, if there exists p such that $p = \phi(T, p)$ for some *T*. In a phase space of dimension two, a periodic solution constitutes a closed orbit, which is called a limit cycle.

A periodic solution curve Γ is stable for any $\varepsilon > 0$, if there exists a neighbourhood U of Γ and $d(\phi(t, x), \Gamma) < \varepsilon$, for any $x \in U$ and $t \geq 0$. For asymptotic stability or asymptotic convergence of a periodic solution Γ, the followings should be satisfied for all x that is contained in the neighbourhood U of Γ.

$$\lim_{t \to +\infty} d(\phi(t, x), \Gamma) = 0 \qquad (2.4)$$

The Poincare-Bendixson theorem is most often used to establish the sufficient conditions for the existence of closed orbits in the dynamical system of (2.3), although it is of limited use for two-dimensional systems (see Lorentz (1993)).

So far, the discussion has been limited to continuous time nonlinear dynamical systems, but this is only for a reference frame of the non-linear time series model in discrete time.

A limit cycle in discrete time is defined as follows (see Tong and Lim (1980)). Let x_n denote a $k \times 1$ dimensional state that satisfies the equation

$$x_n = f(x_{n-1}) \qquad (2.5).$$

A limit point in the vector space is defined as follows: A $k \times 1$ dimensional vector x^* is called a limit point, if there exists $x_0 \neq x^*$ such that, starting with $n = 0$, x_n tends to x^* as n tends to infinity: $\lim_{n \to +\infty} x_n = x^*$.

A limit cycle in discrete time is defined as follows: Let C denote the set of $k \times 1$ dimensional vector c_i, $i = 1, \cdots, T, T$ being a positive integer $\leq \infty$. 1) C is called a limit cycle of period T if there exists $x_0 \notin C$, such that as $n \to +\infty$, x_n will ultimately fall into $x^* : x_n \to x^*$, 2) $c_i = f(c_{i-1})$ $i = 2, 3, \ldots, T$, $c_{T+i} = c_i$ $i = 1, 2, \ldots, T$, and 3) T is the smallest such positive integer.

On the effectiveness of the TAR model in inferring the existence of limit cycles in time series, the following notation by Tong (1983) is instructive:

> "Physically, limit cycles represent the stationary state of sustained oscillations (now dynamic) which do not depend on initial conditions but depend exclusively on the parameters of the system, i.e. they are intrinsic properties. In addition, there exist limit cycles which have the properties of being robust, i.e. insensitive to small perturbation of parameters of the system."

3. Linearity Tests

In the preceding section, it was shown that the class of threshold autoregressive models can capture the notion of a limit cycle. In this section, the method of linearity testing, which checks whether there are limit cycles in macroeconomic variables in Japan, is briefly explained.

Following Hansen (1999), we show the model that tests the linearity of time series data. The threshold autoregressive (TAR) model is a nonlinear model that consists of several regimes, each of which constructs an autoregressive model that is linear. The variable that decides in which regime each observation is contained is named a threshold variable. Among the several classes of TAR models, the TAR model in which the threshold variable is its own lag is named the 'Self Exciting Threshold Autoregressive' (SETAR) model.

A SETAR(m) model, in which there are m regimes, takes the form

$$y_t = \alpha_1' X_{t-1} I_{1t}(\gamma, d) + \cdots + \alpha_m' X_{t-1} I_{mt}(\gamma, d) + e_t \qquad (3.1) ,$$

where $X_{t-1} = (1, y_{t-1}, y_{t-2}, \ldots, y_{t-p})'$, which has $k \times 1$ vector ($k = 1 + p$), $\gamma = (\gamma_1, \ldots, \gamma_{m-1})$, ($\gamma_1 < \gamma_2 < \cdots < \gamma_{m-1}$), which is named a threshold value to group the observations into regimes. $I_{jt}(\gamma, d) = I(\gamma_{j-1} < y_{t-d} \leq \gamma_j)$ are dummy variables, and are 1 when $\gamma_{j-1} < y_{t-d} \leq \gamma_j$, and 0 otherwise. When the observations are in $\gamma_{j-1} < y_{t-d} \leq \gamma_j$, the processes of the time series are in accordance with AR(p) process $Y_t = \alpha_j' X_{t-1}$.

The parameters of (1) may be collected as $\theta = (\alpha_1, \alpha_2, \ldots, \alpha_m, \gamma, d)$. The parameters are estimated by least-squares (LS), given (γ, d). (γ, d) are determined by minimizing the sum of the squared residuals of (1).

The testing for linearity is to test the null hypothesis of SETAR(1) against the alternative of SETAR(2). The SETAR(1) model can be written as

$$y_t = \alpha_1' X_{t-1} + e_t \qquad (3.2) ,$$

which is an AR model, and SETAR(2) as

$$y_t = \alpha_1' X_{t-1} I_{1t}(\gamma, d) + \alpha_2' X_{t-1} I_{2t}(\gamma, d) + e_t \qquad (3.3).$$

The statistic to test the linearity against the TAR model is $F_{12} = n\left(\frac{S_1 - S_2}{S_2}\right)$, where S_m ($m = 1,2$) is the sum of squared residuals of the m-regime TAR model. If the threshold value γ and delay parameter d are fixed, F_{12} is distributed to $\chi^2(k)$ under the null hypothesis of SETAR(1) which is actually an AR model. However, the asymptotic distribution of F_{12} is not χ^2, unless the threshold value γ or delay parameter d are identified. The distribution is estimated by the bootstrap method. If the value of F_{12} is larger than the critical value of $\chi^2(k)$, i.e. if the linearity is rejected, the SETAR(2) model is supported.

When the SETAR(2) model is adopted, we proceed to test whether the series are subject to SETAR(2) or SETAR(3) models. The SETAR(3) model can be written as

$$y_t = \alpha_1' X_{t-1} I_{1t}(\gamma, d) + \alpha_2' X_{t-1} I_{2t}(\gamma, d) + \alpha_3' X_{t-1} I_{3t}(\gamma, d) + e_t \qquad (3.4),$$

where $\gamma = (\gamma_1, \gamma_2)$, because there are two threshold values in the three-regime TAR model. If the threshold value γ and delay parameter d are fixed, the test statistic is $F_{23} = n\left(\frac{S_2 - S_3}{S_3}\right)$[2], which is also distributed to $\chi^2(k)$ under the null hypothesis of the SETAR(2) model. Otherwise, the distribution of F_{23} is not χ^2. The distribution is estimated by the bootstrap method. If the value of F_{23} is larger than the critical value of $\chi^2(k)$, the SETAR(2) model is rejected, and the SETAR(3) model is adopted.

4. An Illustration of the Simulated TAR Process

In this section, we illustrate examples of simulated AR and TAR models that establish the characteristics of the TAR models to be compared to an AR model. First, a two-regime TAR model is examined, and then a three-regime TAR model.

[2] The notation of F_{23} is the same as that of F_{12}.

The data processes generated from the AR(1) and SETAR(2;1,1)[3] models are plotted in Figure 1. The AR(1) model is as follows:

$$y_t = 0.7 y_{t-1} + \varepsilon_t \qquad (4.1),$$

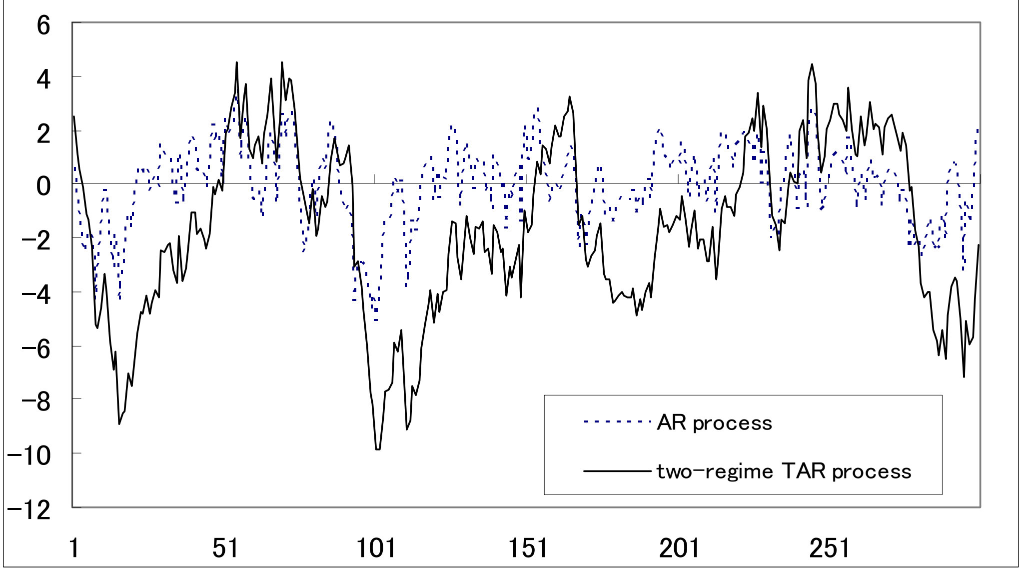
where the error terms are generated from $\varepsilon_t \sim$ i.i.d.N(0,1). The two-regime TAR (SETAR(2;1,1)) model is as follows:

$$y_t = \begin{cases} -0.3 + 0.9 y_{t-1} + \varepsilon_t, & y_{t-1} \le 0 \\ 1 + 0.5 y_{t-1} + \varepsilon_t, & y_{t-1} > 0 \end{cases}, \qquad (4.2)$$

where ε_t is also distributed as i.i.d.N(0,1), and the threshold value is zero. The error terms in both the AR(1) and SETAR(2;1,1) models are generated from the same random variable.

While the coefficient of y_{t-1} in the SETAR(2;1,1) model is 0.9 in regime 1 ($y_{t-1} \le 0$), in regime 2 ($y_{t-1} > 0$) it is 0.5. The threshold value is zero as mentioned above. This means that the speed of return to zero in regime 1 ($y_{t-1} \le 0$) is slow, while that in regime 2 ($y_{t-1} > 0$) is fast. As a result, the data process generated from the two-regime TAR model has a higher probability of the observations being grouped into regime 1than into regime 2. Moreover, we can see from Figure 1 that the data from the SETAR(2;1,1) process are more persistent than those from the AR(1) process.

Figure 1: AR and two-regime TAR processes



[3] SETAR(2;1,1) means that the SETAR model has two regimes, each of which constitutes an AR(1) model.

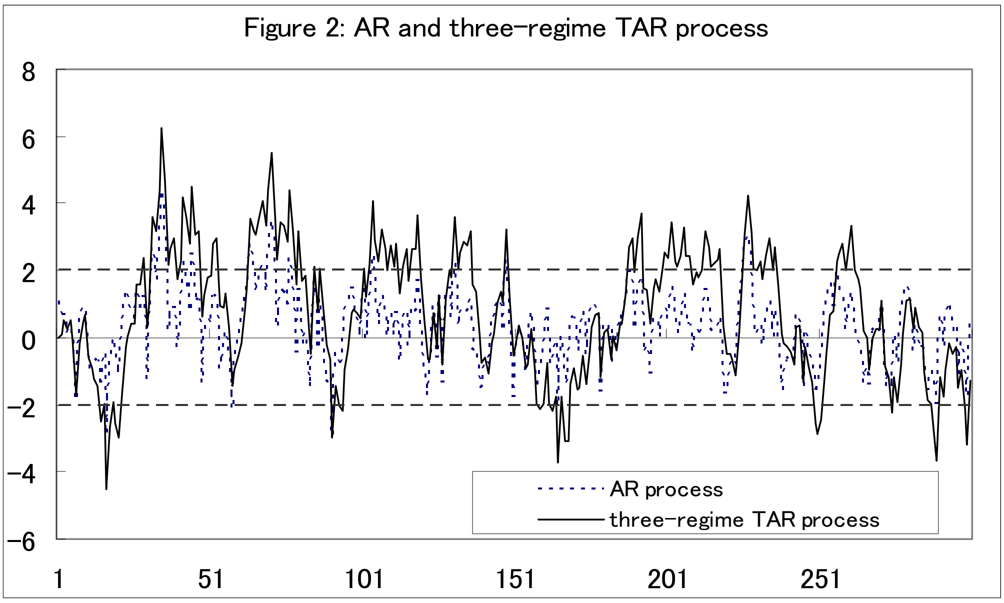
Next, we examine the data generated from a three-regime TAR (SETAR(3;1,1,1)) process by comparing it with the data from the AR(1) process. These data are plotted in Figure 2. The AR(1) model is as follows:

$$y_t = 0.5 y_{t-1} + \varepsilon_t \qquad (4.3),$$

where the error terms are generated as $\varepsilon_t \sim$ i.i.d.N(0,1). The three-regime TAR (SETAR(3;1,1,1)) model is as follows:

$$y_t = \begin{cases} -1 + 0.5 y_{t-1} + \varepsilon_t, & y_{t-1} \le -2 \\ 0.95 y_{t-1} + \varepsilon_t, & -2 < y_{t-1} \le 2, \\ 1 + 0.5 y_{t-1} + \varepsilon_t, & y_{t-1} > 2 \end{cases} \qquad (4.4)$$

where ε_t is also distributed as i.i.d.N(0,1), and the threshold values are set to ± 2. The error terms in both the AR(1) and SETAR(3;1,1,1) models are also generated from the same random variable.



The coefficient of y_{t-1} in the AR(1) process here is 0.5 and it is less than that in Figure 1, whose coefficient is 0.7. It is found that the AR(1) process in Figure 1 is more persistent than the AR(1) process in Figure 2.

In the SETAR(3;1,1,1) model, we set the coefficient parameters on y_{t-1} to 0.5 in both regime 1 ($y_{t-1} \le -2$) and regime 3 ($y_{t-1} > 2$), and to 0.95 in regime 2 ($-2 < y_{t-1} \le 2$). When the variable y_t enters regime 2, it tends to remain in the same regime persistently. On the other hand, the process easily tends to come near ± 2 in regime 1 or regime 3, because the value of the coefficient in y_{t-1} is 0.5. We can easily see that most observations of the SETAR model are in regime 2 in Figure 2.

5. Empirical Results and the Implications

5.1 Data and Basic Statistics

The macroeconomic variables in Japan examined in this paper are listed in Table 1. Composite Index (CI) and Cumulated Diffusion Index (DI) are taken as the business cycle index; Consumer Price Index (CPI) and Wholesale Price Index (WPI) as the price index; NIKKEI and TOPIX as the stock price index; M1 and M2 as money supply; and the return of a 10-year Government Bond (LR) as the interest rate. We examine monthly data, because we cannot detect nonlinearity from quarterly data. Because the original level values of all variables are not stationary, one-month differences of the natural log values are taken for our analysis. There is large irregular fluctuation in each variable, because the series are one-month difference, not one-year difference.

Table 1

Name	Data	from	to
CI	Composite Index (Coincident Series)	1980:2	2006:8
DI	Cumulated Diffusion Index (Coincident Series)	1957:1	2006:8
CPI	Consumer Price Index	1970:2	2006:8
WPI	Wholesale Price Index	1960:2	2006:9
Nikkei225	Nikkei Average Stock Price	1951:2	2006:9
TOPIX	Tosho Stock Price Index	1955:2	2006:9
M2	M2+CD Seasonal Adjusted	1955:2	2006:8
M1	M1Seasonal Adjusted	1955:2	2006:8
10yearbond	Rate of Return on Government Bond (10years)	1985:1	2006:10

The basic statistics of both the level and difference variables are shown in Table 2. In the log difference series, the fluctuations in the stock indices, i.e. NIKKEI and TOPIX, are very large, and the stock indices and money supply indices, i.e. M2 and M1, have strong upward trends. WPI and LR have higher values of kurtosis, and WPI also has a positive high value of skewness.

Table 2

(1) Level

	Observations	Mean	Std Dev	Minimum	Maximum	Skewness	Kurtosis
DI	597	5,731.70	1,980.03	1,070.00	8,393.60	-0.838	-0.447
CI	320	95.37	8.60	80.50	113.10	0.242	-0.936
CPI	440	81.28	21.23	31.10	101.80	-1.103	-0.017
WPI	561	88.09	23.74	48.90	116.10	-0.652	-1.230
NIKKEI	669	8,966.50	8,744.56	108.48	38,130.00	0.990	0.210
TOPIX	621	778.02	696.10	30.42	2,859.57	0.730	-0.513
M2	620	2,762,891.66	2,477,073.35	35,645	7,180,962	0.421	-1.368
M1	620	985,349.59	1,058,668.96	17,188	3,987,296	1.373	1.063
LR	262	3.46	1.96	0.47	7.79	0.267	-1.348

(2) Logged first difference

	Observations	Mean	Std Dev	Minimum	Maximum	Skewness	Kurtosis
DI	596	0.322	0.998	-4.567	4.567	0.154	5.411
CI	319	0.082	0.933	-2.534	2.652	-0.194	-0.229
CPI	439	0.264	0.675	-1.105	4.101	1.644	4.776
WPI	560	0.128	0.597	-1.032	7.170	4.909	43.493
NIKKEI	668	0.747	4.569	-18.380	14.972	-0.465	1.508
TOPIX	620	0.634	4.109	-14.623	13.360	-0.310	0.603
M2	619	0.857	0.675	-1.612	3.113	-0.020	0.092
M1	619	0.878	1.455	-7.183	9.405	-0.178	4.772
LR	261	-0.516	10.464	-52.235	71.940	1.975	16.922

5.2 Results for the Linearity Test

The results for the two-regime TAR model test are presented in Table 3(1). P-values are calculated by the bootstrap method. Linearity is rejected for all variables except CI, NIKKEI and LR. It turns out that the processes of DI, CPI, WPI, TOPIX, M2, and M1 are subject to the TAR model. Therefore, it is suggested that they have limit cycles.

The variables, which in the previous test turned out to be subjected to two-regime TAR, should be examined, whether they are subject to the two-regime TAR or three-regime TAR model. The results of the test as to whether each variable is subject to the two-regime or three-regime TAR models are shown in Table 3(2). The null hypothesis in the cases of CPI, WPI and TOPIX is rejected at a 5% significance level, and that of M1 is rejected at a 10% significance level. It turns out that many variables are subject to the three-regime TAR model.

Table 3: Linearity tests

(1) H_0 :AR Model vs H_1 : Two-regime TAR test

	CI	DI	CPI	WPI	NIKKEI
P-value	0.223	0.042	0.018	0.001	0.233
	TOPIX	M2	M1	LR	
P-value	0.016	0.041	0.046	0.554	

(2) H_0 : Two-regime TAR vs H_1 : Three-regime TAR test

	DI	CPI	WPI	TOPIX	M1	M2
P-value	0.205	0.045	0.008	0.016	0.112	0.070

* Monthly data are used, and are transformed into log-difference, i.e. $\{\ln(y_t) - \ln(y_{t-1})\} \times 100$. P-values are calculated by bootstrap method, considering heteroscedasticity when it is observed in the error terms. The lag length is selected by BIC criteria.

5.3 Some Implications from the Results

We now discuss some implications of the estimated TAR models for data processes. The two-regime TAR estimation results for DI, CPI, WPI, TOPIX, M2 and M1, which turned out to be subject to the two-regime TAR model rather than the linear AR model, are presented in Table 4. The three-regime estimation results for CPI, WPI, TOPIX and M2, which turned out to be subject to the three-regime TAR model, are presented in Table 5. We examine the estimated results from the two-regime TAR models on the DI, M1 and M2 data processes, and those from the three-regime TAR models on the CPI, WPI, TOPIX and M1 data processes.

Table4：Two-regime TAR　estimates (1)

	DI		CPI	
	Regime 1	Regime 2	Regime 1	Regime 2
	$y_{t-6} \le -0.541$	$y_{t-6} > -0.541$	$y_{t-12} \le 0.803$	$y_{t-12} > 0.803$
μ_i	-0.131(0.143)	0.025(0.018)	-0.009(0.024)	0.905(0.183)
ϕ_{i1}	0.996(0.253)	0.492(0.093)	0.162(0.088)	0.103(0.086)
ϕ_{i2}	-0.706(0.302)	0.324(0.083)	-0.181(0.055)	0.050(0.109)
ϕ_{i3}	0.651(0.279)	-0.096(0.058)	0.003(0.054)	0.136(0.120)
ϕ_{i4}	-0.346(0.242)	0.275(0.052)	0.052(0.065)	-0.063(0.096)
ϕ_{i5}	0.188(0.203)	0.019(0.050)	0.089(0.051)	0.281(0.095)
ϕ_{i6}	0.067(0.133)	-0.102(0.048)	0.160(0.051)	-0.251(0.105)
ϕ_{i7}	-0.526(0.274)	0.046(0.034)	0.022(0.061)	0.305(0.098)
ϕ_{i8}	-0.024(0.245)	-0.092(0.033)	0.155(0.055)	-0.323(0.080)
ϕ_{i9}	0.709(0.223)	0.021(0.044)	0.060(0.048)	0.305(0.098)
ϕ_{i10}	-0.700(0.240)	-0.020(0.040)	0.006(0.055)	-0.509(0.107)
ϕ_{i11}	---	---	0.019(0.044)	0.414(0.098)
ϕ_{i12}	---	---	0.507(0.069)	-0.191(0.133)
$\sum_j \phi_{ij}$	0.309	0.867	1.054	0.227
P_i	0.123	0.877	0.838	0.162
SM_i	-0.298	-0.444	0.837	1.087

	WPI		TOPIX	
	Regime 1	Regime 2	Regime 1	Regime 2
	$y_{t-2} \le 0.585$	$y_{t-2} > 0.585$	$y_{t-1} \le -3.562$	$y_{t-1} > -3.562$
μ_i	0.016(0.013)	0.437(0.095)	-4.241(1.495)	0.616(0.193)
ϕ_{i1}	0.316(0.053)	0.828(0.156)	-0.316(0.258)	0.294(0.057)
ϕ_{i2}	0.139(0.058)	-0.314(0.110)	---	---
ϕ_{i3}	0.085(0.053)	0.085(0.131)	---	---
ϕ_{i4}	0.155(0.045)	0.035(0.119)	---	---
$\sum_j \phi_{ij}$	0.695	0.634	-0.316	0.294
P_i	0.895	0.105	0.147	0.853
SM_i	0.423	0.808	-3.115	-0.431

Table4：Two-regime TAR estimates(2)

	M2		M1	
	Regime 1	Regime 2	Regime 1	Regime 2
	$y_{t-3} \le 1.388$	$y_{t-3} > 1.388$	$y_{t-1} \le 1.099$	$y_{t-1} > 1.099$
μ_i	0.030(0.033)	0.746(0.234)	0.426(0.112)	0.628(0.348)
ϕ_{i1}	-0.182(0.074)	0.005(0.118)	-0.367(0.092)	-0.248(0.172)
ϕ_{i2}	0.060(0.047)	0.253(0.095)	-0.134(0.073)	0.099(0.075)
ϕ_{i3}	0.301(0.047)	0.317(0.130)	0.019(0.056)	0.484(0.078)
ϕ_{i4}	0.196(0.042)	0.138(0.089)	0.019(0.049)	0.199(0.066)
ϕ_{i5}	0.215(0.047)	-0.041(0.101)	0.126(0.048)	0.085(0.054)
ϕ_{i6}	0.260(0.050)	-0.092(0.101)	0.205(0.055)	0.039(0.075)
ϕ_{i7}	0.084(0.049)	-0.084(0.108)	0.073(0.054)	-0.002(0.065)
ϕ_{i8}	---	---	-0.027(0.049)	0.025(0.049)
ϕ_{i9}	---	---	0.112(0.041)	0.010(0.056)
ϕ_{i10}	---	---	-0.039(0.036)	0.016(0.087)
ϕ_{i11}	---	---	0.121(0.048)	0.025(0.078)
ϕ_{i12}	---	---	0.048(0.058)	-0.194(0.074)
$\sum_j \phi_{ij}$	0.935	0.496	0.156	0.538
P_i	0.790	0.210	0.573	0.427
SM_i	1.328	1.452	0.597	1.219

The standard deviations are shown in parentheses. P_i presents the ratio in each regime. The estimated models are as follows:

$$y_t = \begin{cases} \mu_1 + \phi_{11} y_{t-1} + \cdots + \phi_{1p} y_{t-p} + \varepsilon_{1t}, & if \quad y_{t-d} \le \gamma \\ \mu_2 + \phi_{21} y_{t-1} + \cdots + \phi_{2p} y_{t-p} + \varepsilon_{2t}, & if \quad y_{t-d} > \gamma \end{cases}$$

where ε_{it} is the error term in each regime. γ is the threshold value.

$$SM_i = \mu_i + \gamma \times \sum \phi_{ij}$$

In the last row of the estimated results in Table 4 and Table 5, we show the index SM_i, which takes into account whether the process tends to remain in the same regime or to move to another regime. The index is calculated by $SM_i = \mu_i + \gamma \times \sum \phi_{ij}$, where i refers to the regime, and γ denotes the threshold value. We refer the index to the SM (Stop-or-Move

Index) below. While the SM index calculates the total effect from the constant term plus all explanatory variables at the average of two threshold values in regime 2 of the three-regime TAR model, in the other regimes it calculates the total effect from the constant term plus all explanatory variables at the threshold value. When the value of the explained variable is in one regime, the SM index checks whether the process tends to remain in the same regime or to move to another regime.

We found several patterns in the two-regime TAR model in the processes DI, M2 and M1. In the two-regime TAR model, regime 1 covers $y_{t-d} \leq \gamma$, while regime 2 covers $\gamma < y_{t-d}$. The DI process tends to move from regime 1 to regime 2, because the value of SM_1 in regime 1 is –0.298, and is greater than the threshold value, –0.541. Therefore, we guess that the DI data in regime 1 tend to move to regime 2. On the other hand, the DI data in regime 2 tend to remain in the same regime, because the value of SM_2 in regime 2 is –0.444, which is greater than the threshold value, –0.541. As a consequence, the ratio (P_i) that the variable is in regime 2 is 88%. The M2 process tends to move to the threshold value in both regime 1 and regime 2, because the values of SM_1 and SM_2 are 1.328 and 1.452, respectively, which are almost the same as the threshold value, 1.388. The ratios in regime 1 and regime 2 are almost 80% and 20%, respectively. Finally, the M1 process tends to remain in the same regime in both regimes, because the value of SM_1 is 0.597, which is less than the threshold value, 1.1, while the value of SM_2 is 1.2, which is greater than the threshold value. The ratios in regime 1 and regime 2 are 57% and 43%, respectively. Although the process of the three variables is in accordance with the two-regime TAR model, the character of each process is different.

Although the processes of CPI, WPI, TOPIX and M1 are subject to the three-regime TAR model, the pattern of each process differs slightly. The processes of CPI and WPI tend to remain in the same regime in all three regimes. The SM_1 value of CPI is 0.513, which is slightly lower than the smaller threshold value 0.52, the SM_2 value is 0.612, which is between the smaller threshold value 0.52 and the larger one 0.81, and the SM_3 value is 1.13, which is greater than the larger threshold value. The SM_1 value of WPI is clearly lower than the smaller threshold value, the SM_2 value is between two threshold value, and the SM_3 value is greater than the larger threshold value. On the other hand, the TOPIX process tends to move from regime 2 to regime 1, because the SM_1 value is –0.296, which is in regime 2, while the data in regime 3 tend to remain in regime 3. The M1 process tends to move strongly from regime 1 to regime 2. Therefore, we conclude that M1 is generated by the two-regime TAR model, although the process is rejected against the two-regime TAR model at a 10% significance level. As stated above, the character of each process in the three-regime TAR model also differs from that of the others[4].

Table 5 : Three-regime TAR estimates(1)

	CPI		
	Regime 1	Regime 2	Regime 3
	$y_{t-12} \leq 0.520$	$0.520 < y_{t-12} \leq 0.808$	$y_{t-12} > 0.808$
μ_i	0.003(0.021)	-0.239(0.799)	0.958(0.191)
ϕ_{i1}	0.081(0.060)	0.496(0.238)	0.113(0.088)
ϕ_{i2}	-0.175(0.061)	-0.171(0.211)	0.014(0.107)
ϕ_{i3}	0.021(0.055)	-0.118(0.105)	0.141(0.118)
ϕ_{i4}	-0.031(0.049)	0.449(0.195)	-0.077(0.097)
ϕ_{i5}	0.103(0.052)	0.011(0.112)	0.316(0.094)
ϕ_{i6}	0.088(0.051)	0.541(0.111)	-0.285(0.103)
ϕ_{i7}	0.070(0.051)	-0.260(0.181)	0.280(0.088)
ϕ_{i8}	0.138(0.055)	0.099(0.126)	-0.343(0.080)
ϕ_{i9}	0.081(0.054)	-0.048(0.112)	0.309(0.094)
ϕ_{i10}	0.018(0.047)	0.024(0.124)	-0.464(0.113)
ϕ_{i11}	0.056(0.051)	-0.189(0.099)	0.426(0.098)
ϕ_{i12}	0.531(0.071)	0.448(1.149)	-0.216(0.132)
$\sum_j \phi_{ij}$	0.981	1.282	0.214
P_i	0.742	0.101	0.157
SM_i	0.513	0.612	1.131

[4] There would be a few economic factors to be related to the behaviour of the processes to be examined in this paper. As comments at JEPA conference, T. Matsuki at Osaka Gakuin University pointed out mean shifts in M2, and sudden jumps in CPI and WPI as the factors. We appreciate his excellent comments and suggestions. We will need to carefully examine the relationship in our next paper.

	WPI		
	Regime 1	Regime 2	Regime 3
	$y_{t-2} \le 0.096$	$0.096 < y_{t-2} \le 0.588$	$y_{t-2} > 0.588$
μ_i	-0.014(0.017)	0.186(0.081)	0.443(0.098)
ϕ_{i1}	0.294(0.060)	0.396(0.238)	0.829(0.156)
ϕ_{i2}	0.058(0.076)	-0.394(0.245)	-0.316(0.110)
ϕ_{i3}	0.087(0.062)	0.095(0.083)	0.086(0.131)
ϕ_{i4}	0.108(0.056)	0.199(0.066)	0.033(0.119)
$\sum_j \phi_{ij}$	0.547	0.296	0.632
P_i	0.633	0.264	0.103
SM_i	0.039	0.287	0.815

Table 5 : Three-regime TAR estimates(2)

	TOPIX		
	Regime 1	Regime 2	Regime 3
	$y_{t-1} \le -3.505$	$-3.505 < y_{t-1} \le -0.069$	$y_{t-1} > -0.069$
μ_i	-4.703(1.446)	-0.483(0.501)	0.993(0.318)
ϕ_{i1}	-0.296(0.252)	-0.258(0.316)	0.222(0.084)
$\sum_j \phi_{ij}$	-0.296	-0.258	0.222
P_i	0.153	0.268	0.579
SM_i	-3.666	-0.468	0.978

M1			
	Regime 1	Regime 2	Regime 3
	$y_{t-1} \le -0.231$	$-0.231 < y_{t-1} \le 1.096$	$y_{t-1} > 1.096$
μ_i	0.166(0.258)	0.534(0.153)	0.627(0.347)
ϕ_{i1}	-0.550(0.148)	-0.233(0.223)	-0.247(0.171)
ϕ_{i2}	-0.178(0.109)	-0.146(0.095)	0.099(0.075)
ϕ_{i3}	0.108(0.072)	-0.050(0.066)	0.484(0.078)
ϕ_{i4}	0.124(0.055)	-0.113(0.075)	0.200(0.066)
ϕ_{i5}	0.130(0.068)	0.164(0.061)	0.085(0.054)
ϕ_{i6}	0.032(0.061)	0.296(0.084)	0.040(0.074)
ϕ_{i7}	0.082(0.049)	0.095(0.082)	-0.001(0.065)
ϕ_{i8}	-0.143(0.085)	0.027(0.055)	0.025(0.050)
ϕ_{i9}	0.093(0.062)	0.099(0.049)	0.011(0.056)
ϕ_{i10}	-0.042(0.049)	-0.073(0.046)	0.016(0.087)
ϕ_{i11}	0.094(0.083)	0.134(0.047)	0.025(0.077)
ϕ_{i12}	0.285(0.091)	-0.082(0.056)	-0.195(0.073)
$\sum_j \phi_{ij}$	0.035	0.118	0.542
P_i	0.166	0.405	0.429
SM_i	0.158	0.585	1.221

The estimated models are as follows:

$$y_t = \begin{cases} \mu_1 + \phi_{11} y_{t-1} + \cdots + \phi_{1p} y_{t-1} + \varepsilon_{1t} & if \quad y_{t-d} \le \gamma_1 \\ \mu_2 + \phi_{21} y_{t-1} + \cdots + \phi_{2p} y_{t-p} + \varepsilon_{2t} & if \quad \gamma_1 < y_{t-d} \le \gamma_2 \\ \mu_3 + \phi_{31} y_{t-1} + \cdots + \phi_{3p} y_{t-p} + \varepsilon_{3t} & if \quad y_{t-d} > \gamma_2 \end{cases}$$

where γ_1 and γ_2 are threshold values.

$$SM_i = \mu_i + \gamma \times \sum \phi_{ij} \text{ ,where } \gamma = \begin{cases} \gamma_1 & if \quad regime\ 1\ (y_{t-d} \le \gamma_1) \\ (\gamma_1 + \gamma_2)/2 & if \quad regime\ 2\ (\gamma_1 < y_{t-d} \le \gamma_2) \\ \gamma_2 & if \quad regime\ 3\ (y_{t-d} > \gamma_2) \end{cases}$$

6. Conclusions

In this paper, we examined whether macroeconomic variables in Japan have limit cycles, and are therefore subject to TAR models, by testing the linearity. We found that six of the nine series are subject to TAR models, and that there are limit cycles in their Japanese macroeconomic variables. The results indicate that the behaviour of these variables is generated by endogenous business cycle theory, and this implies that the government could control the business cycle in Japan, even though the cyclical behaviour is extremely complicated.

We also tested whether the processes of the variables, which are rejected for linearity, follow the two-regime or three-regime TAR models. As a result of the test, we conclude that CPI, WPI and TOPIX follow the three-regime TAR model. We may also have found that each process has a unique behaviour.

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