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MARKETS IN LICENSES REVISITED: IMPLICATIONS FOR CLIMATE POLICY

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ABSTRACT

The idea of introducing "markets in licenses" to environmental quality control policy was first suggested with a rigorous economic foundation by Montgomery (JET: 1972; "Markets in Licenses and Efficient Pollution Control Programs"). This well-known classic paper considered regional environmental problems: There exist *m* industrial sources of pollution; each source emits a single pollutant affecting the environmental quality of several (*n*) locations; and the standard of environmental quality is chosen as a goal by a resource management agency. To develop decentralized systems for achieving environmental goals at multiple locations, Montgomery addressed two types of markets in licenses. One is *the market in licenses to pollute*, and the other is *the market in emission licenses*.

Montgomery's study greatly contributed to the academic literature by offering a theoretical basis for subsequent emissions trading debates. On the other hand, the problem he considered was regional area-wide environmental problems, and thus his model setting seemed too general when used to address climate change. Some may even consider it obsolete.

However, when we step back to look at the challenges of climate change in a more general perspective, we must realize that Montgomery's model is widely applicable to this issue.

The purpose of this study is to reconsider the relations between the two markets in licenses using the same framework as Montgomery, to show that his framework is still insightful. The emphasis is upon whether these two markets are compatible, and if so, in what conditions. These questions have never been answered by Montgomery's original work. Although the result obtained here may seem abstract, it has important implications for future climate change policy debates, such as the post-Kyoto debates. It also suggests that current emissions market regimes, such as EU-ETS and the Kyoto mechanisms, may not be sufficient for the purpose of mitigating climate change, and that some additional technology policies are needed.

Key words: Emissions trading; Property rights; Climate change.

JEL Classification: Q58, D23, H11.

MARKETS IN LICENSES REVISITED: IMPLICATIONS FOR CLIMATE POLICY

1. Introduction

The idea of introducing “markets in licenses” to environmental quality control policy was first suggested with a rigorous economic foundation by Montgomery (1972). This well-known classic paper considered regional environmental problems: There exist *m* industrial sources of pollution, each of which is fixed in location, and is owned by an independent, profit-maximizing firm; each source emits a single pollutant, which causes pollutant concentrations at several (*n*) locations, affecting the environmental quality of these locations; and the standard of environmental quality is chosen as a goal by a resource management agency. The relationship between emissions from the source locations and pollution levels at the regulation locations is represented as a matrix of unit diffusion coefficients. To develop decentralized systems for achieving environmental goals at the multiple locations, Montgomery addressed two types of markets in licenses. One is *the market in licenses to pollute*, and the other is *the market in emission licenses*.

Montgomery’s study greatly contributed to the academic literature by presenting a theoretical basis for emissions trading debates thereafter. On the other hand, the problem he considered was regional, area-wide environmental problems, and thus his model setting seems too general to address climate change: as a global warming phenomenon, it is enough for us to consider a single pollution location (i.e. $n = 1$). With such a setting, working on two different types of markets in licenses does not seem to provide a useful case study, although it is an intellectually interesting exercise. In this sense, Montgomery’s model may seem obsolete.

However, when we step back to look at our challenges of climate change from a more general perspective, we may realize that Montgomery’s model is widely applicable to this issue. For example, let us consider *T* years to be under regulation. Let annual greenhouse gas (GHG) emissions correspond to “emission locations” in the Montgomery model. Then, the annual levels of GHG net accumulation (or concentration) in the atmosphere can be represented as “regulation locations” in the Montgomery model.

Similarly, several ways of applying his model are also possible in a static framework. One possibility is to consider “regulation locations” in the Montgomery model as indexes for impacts, adaptation, and vulnerability. The scientific relations between emissions and impacts, adaptation, and vulnerability are among the important topics in the United Nation’s Intergovernmental Panel on Climate Change (IPCC) Working Group 2.

In any case, any emissions trading regimes under the current debates – either the EU Emissions Trading Scheme (EU-ETS) or the Kyoto mechanisms – correspond to *the market in emission licenses* in Montgomery's model.

What was our true goal in the climate change debate? The fundamental cause of the greenhouse effect is GHG concentration in the atmosphere. The reason that the effect is problematic is that it has a great impact on nature, and thus we need to seek adaptation. Having reflected in this way, we may ask ourselves: Is *the market in emission licenses* truly effective as a climate change mitigation policy? To answer this question, it is beneficial to reconsider Montgomery's model.

The purpose of this study is to reconsider the relations between the two markets in licenses using the same framework as Montgomery. The emphasis is upon whether these two markets are compatible, and if so, in what conditions. Montgomery's original work did not answer these questions in a strict manner. More specifically, this paper analyzes the necessary and sufficient conditions for *the market in emission licenses* to achieve any environmental goals at multiple locations.

The paper is organized as follows. Section 2 introduces Montgomery's diffusion model. Section 3 summarizes his original work. Section 4 sets out the main contribution of this paper, by proposing a fundamental theorem in the same setting as Montgomery. Section 5 applies the result to GHG concentration control policy, and derives two additional theorems. One of these states that *the market in emission licenses* is not always able to replicate the optimal control policies of annual GHG concentrations. The other states that, to provide *the market in emission licenses* with the capability to achieve any annual control policy of GHG concentration, some form of technology policy that helps to accelerate the reduction rate of GHG concentration in the atmosphere is necessary. Section 6 also applies the result of Section 4 to the issues of the impacts of, and adaptation to, climate change. Section 7 concludes the discussion.

2. Montgomery (1972) Diffusion Model

This and the following section briefly introduce the basics of Montgomery's diffusion model. Let us consider a regional environmental problem such that the pollutant given off from an emission source spreads widely, and affects environmental quality at several places. Let i denote an emission source, and let j denote a point of pollution. The number of emission sources is m, and that of pollution points is n, that is, $i = 1$ to m and $j = 1$ to n.[1]

[1] In Montgomery's original setting, m and n are used differently, in that they are replaced by each other. This is simply a matter of preference.

We further assume that h_{ij} (≥0) describes the relation of cause and effect between the emission source i and pollution point j. The overall picture of impact is represented by the following matrix of unit diffusion coefficients.

$$H \equiv \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & h_{ij} & \vdots \\ h_{m1} & \cdots & h_{mn} \end{bmatrix}$$

Let $\boldsymbol{e} \equiv [e_1 \ \cdots \ e_i \ \cdots \ e_m]^T$, $e_i \geq 0 \ \forall i$, denote the levels of emission.[2] By definition, the emission at source i causes pollution $h_{ij}e_i$ at pollution point j. Thus, the total pollution level at point j is described as $\sum_{i=1}^{m} h_{ij}e_i$.

Define the following.

$F_i(e_i)$: Emission reduction cost function for emitter i, and

L^0_j : The target of the level of pollution at point j.

A central planner's problem is described as follows:

$$\underset{\{e_i\}}{Min} \ \sum_{i=1}^{m} F_i(e_i)$$

$$\text{s.t.} \ \sum_{i=1}^{m} h_{ij}e_i \leq L_j^0 \quad \forall j .$$

FONCs are written as follows:

Conditions for Social Optimality (I):

$$F_i'(e_i^S) + \sum_{j=1}^{n} \mu_j^S h_{ij} = 0 \quad \forall i , \tag{2.1}$$

$$L_j^0 - \sum_{i=1}^{m} h_{ij}e_i^S \geq 0, \quad \mu_j^S \geq 0 \ \text{and} \ \mu_j^S \left[L_j^0 - \sum_{i=1}^{m} h_{ij}e_i^S \right] = 0 \quad \forall j . \tag{2.2}$$

$\{\mu_j^S\}$ represents a set of Lagrange multipliers. Superscript "S" indicates that they are socially optimal solutions for the problem.

[2] Superscript T indicates "transpose."

Assuming that each F is decreasing and convex, the above set of conditions is also sufficient. Notice that (2.1) and (2.2) comprise $m + n$ equations containing $m + n$ unknowns. Thus, the solution is unique.

For convenience hereafter, let us assume the following properties of the emission reduction function.

Assumption 2.1

$$F_i'(e_i) \leq 0 \;; \quad F_i''(e_i) > 0 \;; \quad F_i(\bar{e}_i) = 0 \;; \quad F_i'(\bar{e}_i) = 0 \;; \quad \lim_{e_i \to 0+} F_i'(e_i) = -\infty \;\; \forall i.$$

3. Licenses to Pollute

Montgomery explored two types of permits (or licenses) that were intended to implement the socially optimal regime described above. One was *the license to pollute*, and the other was *the emission license*. Let us examine the former in this section.[3]

Let $\{l_{ij}^0\}$ denote the initial allocation of licenses to pollute at pollution point j to emission source i. By definition, we have

$$L_j^0 \equiv \sum_{i=1}^{m} l_{ij}^0 \; .$$

Each emitter's optimization problem is written as follows:

$$\underset{e_i, \{l_{ij}\}_{\forall j}}{Min} \;\; F_i(e_i) + \sum_{j=1}^{n} p_j \cdot \left(l_{ij} - l_{ij}^0\right)$$

$$\text{s.t. } l_{ij} - h_{ij} e_i \geq 0 \quad \forall j \, ,$$

where $\{p_j\}$ represents the market prices of licenses to pollute.

In a competitive market, the following market clearing condition must hold.

$$\sum_{i=1}^{m} \left(l_{ij} - l_{ij}^0\right) \leq 0, \quad p_j \geq 0 \text{ and } p_j \left[\sum_{i=1}^{m} \left(l_{ij} - l_{ij}^0\right)\right] = 0 \quad \forall j \, .$$

Let us introduce an assumption as follows:

[3] In this paper, the two words "license" and "permit" are used interchangeably.

Assumption 3.1 (No private permit creation)

No individual is allowed to create marketable permits (licenses): emitters may only physically buy and sell permits, and the right to issue new permits belongs exclusively to the regulator.

This assumption implies that l_{ij} must be non-negative. This corresponds to a typical condition of Cap-and-Trade systems.

To examine the meaning of this assumption further, suppose, on the contrary, that emitters are allowed to issue permits freely. In other words, suppose that l_{ij} can be negative. Since permits represent the right to pollute, the negative values indicate obligations to clean up pollution on request. This obligation should be securitized, to enable it to be tradable in the market. The security is then traded in the market, and it gives the holder the right to request a pollution clean-up by the originator. The role of the regulator is to verify the validity of the security, and to monitor and confirm the fulfillment of the obligation.

This kind of securitization of the clean-up obligation has already been implemented in the real world. Examples include the project-based mechanisms provided in the Kyoto protocol, namely Joint Implementation (JI) and Clean Development Mechanism (CDM). To obtain credits accruing from JI, also know as Emission Reduction Units (ERUs), one must first make a plan to reduce emissions in a foreign developed country that has ratified the protocol, and is thus obliged to reduce GHG emissions. Then, the plan should be submitted to the United Nations' Framework Convention on Climate Change (UNFCCC) Secretariat. Once the credit is verified and approved by the Secretariat, it is ready for market trading. CDM is similar to JI, in that with this mechanism one can create credits known as Certified Emission Reductions (CERs). Although with CDM the counter party is supposed to be a developing country that has no obligation to undertake emission reduction, the basic idea is similar to JI.

In the remainder of this section, it will be shown that the assumption of no private permit creation is not indispensible for the market in licenses to pollute to work. However, it plays a critical role in the next section. We will return to this point later.

With Assumption 3.1, the market equilibrium is described by the following conditions:

Conditions for the Market in Licenses to Pollute (II):

$$F_i'\left(e_i^*\right) + \sum_{j=1}^{n} \lambda_{ij}^* h_{ij} = 0 \quad \forall i, \tag{3.1}$$

$$p_j^* - \lambda_{ij}^* \geq 0, \quad l_{ij}^* \geq 0 \text{ and } l_{ij}^*\left[p_j^* - \lambda_{ij}^*\right] = 0 \quad \forall i, j\,, \tag{3.2}$$

$$l_{ij}^* - h_{ij}e_i^* \geq 0, \quad \lambda_{ij}^* \geq 0 \text{ and } \lambda_{ij}^*\left[l_{ij}^* - h_{ij}e_i^*\right] = 0 \quad \forall i, j\,, \text{ and} \tag{3.3}$$

$$L_j^0 - \sum_{i=1}^{m} l_{ij}^* \geq 0, \quad p_j^* \geq 0 \text{ and } p_j^*\left[L_j^0 - \sum_{i=1}^{m} l_{ij}^*\right] = 0 \quad \forall j\,, \tag{3.4}$$

where $\{\lambda_{ij}^*\}$ represents Lagrange multipliers. Superscript "*" indicates that they are the equilibrium for this market.

It is easily verified that, given the convexity of the reduction function *F*, the above set of equations has a unique solution. To be more specific, the set of conditions (3.1) to (3.4) comprises *m* + 2 *m n* + *n* equations containing *m* + 2 *m n* + *n* unknowns, $\{e_i^*\}$, $\{p_j^*\}$, $\{l_{ij}^*\}$ and $\{\lambda_{ij}^*\}$.

As noted above, Assumption 3.1 does not play a critical role here. Indeed this assumption is not needed in this model setting. This is addressed as the following lemma.

Lemma 3.1

For all i and j, $l_{ij}^* \geq 0$ *holds in the set of conditions of (II) (i.e.,* $l_{ij}^* < 0$ *never occurs). That is, none of the emitters are willing to create private permits in the market in licenses to pollute.*

Proof

Suppose that for (3.3) and some *i*, there exists *j* such that $l_{ij}^* < 0$ holds. Since $h_{ij} \geq 0$, $e_i^* < 0$ must hold for the selected *i.* However, by Assumption 2.1, we have

$$\lim_{e \to 0+} F_i'(e) = -\infty\,.$$

From (3.1), for the selected *i*, at least one *j*, $\lambda_{ij}^* \to \infty$ must hold. This implies, from (3.2), that $p_j^* \geq \lambda_{ij}^* \to \infty$, meaning that the market explodes, which is a contradiction. (End of proof.)

The following theorem is basically equivalent to "Theorems 3.1 to 3.3" in Montgomery (1972).

Theorem 3.1

The market in licenses to pollute implements a socially optimal distribution of emissions.

Proof

To prove the theorem, it is enough to show the equivalence of conditions (II) with conditions (I). Let us rewrite (II): from Lemma 3.1, the set of conditions of (II) is simplified as follows:

Conditions for the Market in License to Pollute (II-2):

$$F_i'(e_i^*) + \sum_{j=1}^{n} p_j^* h_{ij} = 0 \quad \forall i , \tag{3.5}$$

$$l_{ij}^* - h_{ij} e_i^* \geq 0, \quad p_j^* \geq 0 \text{ and } p_j^*\left[l_{ij}^* - h_{ij} e_i^*\right] = 0 \quad \forall i, j \text{ , and} \tag{3.6}$$

$$L_j^0 - \sum_{i=1}^{m} l_{ij}^* \geq 0, \quad p_j^* \geq 0 \text{ and } p_j^*\left[L_j^0 - \sum_{i=1}^{m} l_{ij}^*\right] = 0 \quad \forall j . \tag{3.7}$$

From (3.6) and (3.7),

$$L_j^0 \geq \sum_{i=1}^{m} l_{ij}^* \geq \sum_{i=1}^{m} h_{ij} e_i^* \text{ , and}$$

$$0 = p_j^*\left[L_j^0 - \sum_{i=1}^{m} l_{ij}^*\right] = p_j^* L_j^0 - p_j^* \sum_{i=1}^{m} l_{ij}^* = p_j^* L_j^0 - p_j^* \sum_{i=1}^{m} h_{ij} e_i^* = p_j^*\left[L_j^0 - \sum_{i=1}^{m} h_{ij} e_i^*\right].$$

Thus (II-2) is further simplified as follows:

$$F_i'(e_i^*) + \sum_{j=1}^{n} p_j^* h_{ij} = 0 \quad \forall i \text{ , and} \tag{3.5}$$

$$L_j^0 - \sum_{i=1}^{m} h_{ij} e_i^* \geq 0, \quad p_j^* \geq 0 \text{ and } p_j^*\left[L_j^0 - \sum_{i=1}^{m} h_{ij} e_i^*\right] = 0 \quad \forall j . \tag{3.8}$$

This set of equations corresponds to those of (I). (End of proof.)

4. Emission Licences

Let us examine the market in emission licenses. Let $\{z_i^k\}$ denote licenses representing the right for Emitter i to emit the pollutant at location k. Also, let $\{z_i^{k0}\}$ denote its initial allocation. The sum of permits issued for location k is defined as follows:

$$Z^{k0} \equiv \sum_{i=1}^{m} z_i^{k0} .$$

Having the set of permits $\{z_i^k\}$, Emitter i faces an emission constraint as follows:

$$e_i \le Min\left\{ \frac{1}{h_{i1}} \sum_{k=1}^{m} h_{k1} z_i^k , \quad \cdots, \quad \frac{1}{h_{ij}} \sum_{k=1}^{m} h_{kj} z_i^k , \quad \cdots, \quad \frac{1}{h_{in}} \sum_{k=1}^{m} h_{kn} z_i^k \right\}$$

Thus, each emitter's optimization problem is written as follows:

$$\underset{e_i}{Min} \;\; F_i(e_i) + \sum_{k=1}^{m} \pi_k \cdot \left(z_i^k - z_i^{k0} \right)$$

$$\text{s.t. } h_{ij} e_i \le \sum_{k=1}^{m} h_{kj} z_i^k \quad \forall j ,$$

where $\{\pi_k\}$ represents a set of market prices for location k.

Market clearing conditions must also hold.

$$\sum_{i=1}^{m} \left(z_i^k - z_i^{k0} \right) \le 0, \quad \pi_k \;\; \ge 0 \text{ and } \pi_k \;\; \cdot \sum_{i=1}^{m} \left(z_i^k - z_i^{k0} \right) = 0 \quad \forall k .$$

Assume again Assumption 3.1. This time, the assumption states that z_{ij} must be non-negative. The market equilibrium for the market is then described by the following conditions.

Conditions for the Market in Emission Licenses (III):

$$F_i'\left(e_i^{**}\right) + \sum_{j=1}^{n} \eta_{ij}^{**} h_{ij} = 0 \quad \forall i , \tag{4.1}$$

$$\pi_k^{**} - \sum_{j=1}^{n} \eta_{ij}^{**} h_{kj} \geq 0, \quad z_i^{k**} \geq 0 \text{ and } z_i^{k**}\left[\pi_k^{**} - \sum_{j=1}^{n} \eta_{ij}^{**} h_{kj}\right] = 0 \quad \forall i, k\,, \tag{4.2}$$

$$\sum_{k=1}^{m} h_{kj} z_i^{k**} - h_{ij} e_i^{**} \geq 0, \quad \eta_{ij}^{**} \geq 0 \text{ and } \eta_{ij}^{**}\left[\sum_{k=1}^{m} h_{kj} z_i^{k**} - h_{ij} e_i^{**}\right] = 0 \quad \forall i, j \text{ , and} \tag{4.3}$$

$$Z^{k0} - \sum_{i=1}^{m} z_i^{k**} \geq 0, \quad \pi_k^{\ **} \geq 0 \text{ and } \pi_k^{\ **}\left[Z^{k0} - \sum_{i=1}^{m} z_i^{k**}\right] = 0 \quad \forall k\,, \tag{4.4}$$

where $\{\eta_{ij}^{**}\}$ represents a set of Lagrange multipliers. Superscript "**" indicates that they are the equilibrium for this market.

The condition $z_i^{k**} \geq 0$ in (4.2) is due to Assumption 3.1. As shown below, this condition plays a critical role in the model. To illustrate this, let us assume another case in which this assumption is removed. With private permit creation allowed, condition (4.2) is replaced by the following:

$$\pi_k^{**} - \sum_{j=1}^{n} \eta_{ij}^{**} h_{kj} = 0 \quad \forall i, k\,. \tag{4.2)'$$

The set of conditions (4.1) to (4.4) constitutes the system of $m + m^2 + m\,n + m$ equations with $m + m + m^2 + m\,n$ unknowns, $\{e_i^{**}\}$, $\{\pi_k^{**}\}$, $\{z_i^{k**}\}$, and $\{\eta_{ij}^{**}\}$. Thus, a unique solution exists. Note that, in Montgomery (1972), the existence of a unique solution is addressed in his "Theorem 3.5," with a rigorous treatment of the above discussion.

Although the set of conditions for the market in emission licenses (III) appears to be similar to that of the conditions for the market in licenses to pollute (II), there are differences. The most fundamental difference is the number of equations that represent marketplaces and trades. Since the right to emit is defined for each emission source, there exist *m* marketplaces. On the other hand, the right to pollute is defined for each pollution point, and thus, there exist *n* marketplaces in that case. These two sets of marketplace may be convertible to each other under certain conditions.

"Theorem 3.6" in Montgomery (1972) basically claims that, if the market for the right to emit can replicate the market for the right to pollute, then cost efficiency holds. However, Montgomery does not clearly address what conditions are necessary as well as sufficient for replication. The next theorem provides such conditions.

Theorem 4.1

The market system with emission licenses attains socially optimal distribution of pollution if and only if the following conditions are satisfied:

(1) Rank (H) = n,

(2) initial allocation of emission license $\{Z^{k0}\}$ *satisfies*

$$L_j^0 = \sum_{k=1}^{m} h_{kj} Z^{k0} \quad \forall j \text{, and}$$

(3) the system allows private creation of emission licenses.

Proof

A key idea of the proof is whether or not, for any set of $\{l_{ij}^*\}$ that satisfies (II), we can construct the set of $\{z_i^{k*}\}$ that satisfies the following condition:

$$l_{ij}^* = \sum_{k=1}^{m} h_{kj} z_i^{k*} \quad \forall i, j \text{ and } z_i^{k*} \geq 0 \quad \forall i, k,$$

or

$$\begin{bmatrix} l_{i1}^* & \cdots & l_{ij}^* & \cdots & l_{in}^* \end{bmatrix} = \begin{bmatrix} z_i^{1*} & \cdots & z_i^{k*} & \cdots & z_i^{m*} \end{bmatrix} H \quad \forall i \text{ and } z_i^{k*} \geq 0 \quad \forall i, k.$$

If Rank (H) = n does not hold, we may not be able to construct such $\{z_i^{k*}\}$ for some $\{l_{ij}^*\}$. Thus, Rank (H) = n is a necessary condition.

If Rank (H) = n holds, then we will be able to construct $\{z_i^{k*}\}$ for any $\{l_{ij}^*\}$, by solving the above set of equations, as follows:

$$\begin{bmatrix} z_i^{1*} & \cdots & z_i^{k*} & \cdots & z_i^{n*} \end{bmatrix} = \begin{bmatrix} l_{i1}^* & \cdots & l_{ij}^* & \cdots & l_{in}^* \end{bmatrix} H_n^{-1} \quad \forall i \text{ and}$$

$$z_i^{k*} = 0 \quad k = n+1, \cdots, m \quad \forall i,$$

where an n by n nonsingular matrix H_n is constructed from n independent rows out of those in matrix H. Note that when Rank (H) = n, we have $m \geq n$.

Notice that the solution does not necessarily satisfy the non-negative condition $z_i^{k*} \geq 0 \quad \forall i, k$. Instead, if we remove the non-negative condition, then the rank condition is sufficient for us to construct $\{z_i^{k*}\}$ for any $\{l_{ij}^*\}$.

Given that a set of $\{z_i^{k*}\}$ is constructed as in the above, the conditions of (3.3) are transformed into the following:

$$\sum_{k=1}^{m} h_{kj} z_i^{k*} - h_{ij} e_i^* \geq 0, \quad \lambda_{ij}^* \geq 0 \text{ and } \lambda_{ij}^* \left[\sum_{k=1}^{m} h_{kj} z_i^{k*} - h_{ij} e_i^* \right] = 0 \quad \forall i, j \,. \tag{3.3)'$$

From the set of $\{p_j^*\}$ that satisfies (II), we can define the following:

$$\pi_k^* = \sum_{j=1}^{n} p_j^* h_{kj} \quad \forall k \,.$$

For the set of $\{\pi_k^*\}$, (3.2) leads to the following:

$$\pi_k^* - \sum_{j=1}^{n} \lambda_{ij}^* h_{kj} \geq 0 \quad \forall i, k \text{ , and} \tag{4.5}$$

$$\begin{aligned} 0 &= \sum_{j=1}^{n} \; l_{ij}^* \left[p_j^* - \lambda_{ij}^* \right] \\ &= \sum_{j=1}^{n} \; \sum_{k=1}^{m} h_{kj} z_i^{k*} \left[p_j^* - \lambda_{ij}^* \right] \\ &= \sum_{k=1}^{m} z_i^{k*} \left[\sum_{j=1}^{n} \left(p_j^* h_{kj} - \lambda_{ij}^* h_{kj} \right) \right] \\ &= \sum_{k=1}^{m} z_i^{k*} \left[\pi_k^* - \sum_{j=1}^{n} \lambda_{ij}^* h_{kj} \right]. \end{aligned} \tag{4.5)'$$

(4.5) and (4.5)' imply that the following equalities must hold:

$$z_i^{k*} \left[\pi_k^* - \sum_{j=1}^{n} \lambda_{ij}^* h_{kj} \right] = 0 \quad \forall i, k \,.$$

Thus, when $z_i^{k*} \geq 0 \quad \forall i, k$, (3.2) is equivalent to

$$\pi_k^* - \sum_{j=1}^{n} \lambda_{ij}^* h_{kj} \geq 0, \quad z_i^{k*} \geq 0 \text{ and } z_i^{k*} \left[\pi_k^* - \sum_{j=1}^{n} \lambda_{ij}^* h_{kj} \right] = 0 \quad \forall i, k \,. \tag{3.2)'$$

From (3.4), we have

$$\sum_{k=1}^{m} h_{kj} Z^{k0} = L_j^0 \geq \sum_{i=1}^{m} l_{ij}^* = \sum_{i=1}^{m} \sum_{k=1}^{m} h_{kj} z_i^{k*} = \sum_{k=1}^{m} h_{kj} \sum_{i=1}^{m} z_i^{k*} .$$

That is

$$\sum_{k=1}^{m} h_{kj} \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) \geq 0 \quad \forall j ,$$

or

$$\left[\left(Z^{10} - \sum_{i=1}^{m} z_i^{1*} \right) \ \cdots \ \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) \ \cdots \ \left(Z^{m0} - \sum_{i=1}^{m} z_i^{m*} \right) \right] H \geq \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} .$$

Note that when Rank (H) = n holds, we can choose n rows that are independent from each other out of matrix H, and construct an n by n nonsingular matrix H_n. Multiplying its inverse matrix from the right, we can obtain the following:

$$Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \geq 0 \quad \forall k .$$

The last part of (3.4) states that

$$p_j^* \cdot \sum_{k=1}^{m} h_{kj} \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) = 0 \quad \forall j .$$

Taking the sum of the above with respect to j, we have the following:

$$0 = \sum_{j=1}^{n} p_j^* \cdot \sum_{k=1}^{m} h_{kj} \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) = \sum_{k=1}^{m} \sum_{j=1}^{n} p_j^* h_{kj} \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) = \sum_{k=1}^{m} \pi_k^* \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) .$$

Since $p_j^* \geq 0$, $h_{ij} \geq 0$, and $\pi_k^* = \sum_{j=1}^{n} p_j^* h_{kj}$, we know $\pi_k^* \geq 0$. Thus,

$$\pi_k^* \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) = 0 \quad \forall k$$

must hold. Then, we finally confirm that (3.4) is equivalent to the following:

$$Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \geq 0, \quad \pi_k^* \geq 0 \text{ and } \pi_k^* \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) = 0 \quad \forall k \;. \tag{3.4)'$$

In summary, $\{e_i^*\}$, $\{p_j^*\}$, $\{l_{ij}^*\}$, and $\{\lambda_{ij}^*\}$ that satisfy conditions (3.1) to (3.4), and $\{\pi_k^*\}$ and $\{z_i^{k*}\}$, which are defined as the above, satisfy the following conditions:

$$F_i'(e_i^*) + \sum_{j=1}^{n} \lambda_{ij}^* h_{ij} = 0 \quad \forall i \,, \tag{3.1)'$$

$$\pi_k^* - \sum_{j=1}^{n} \lambda_{ij}^* h_{kj} \geq 0, \quad z_i^{k*} \geq 0 \text{ and } z_i^{k*} \left[\pi_k^* - \sum_{j=1}^{n} \lambda_{ij}^* h_{kj} \right] = 0 \quad \forall i, k \,, \tag{3.2)'$$

$$\sum_{k=1}^{m} h_{kj} z_i^{k*} - h_{ij} e_i^* \geq 0, \quad \lambda_{ij}^* \geq 0 \text{ and } \lambda_{ij}^* \left[\sum_{k=1}^{m} h_{kj} z_i^{k*} - h_{ij} e_i^* \right] = 0 \quad \forall i, j \text{, and} \tag{3.3)'$$

$$Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \geq 0, \quad \pi_k^* \geq 0 \text{ and } \pi_k^* \left(Z^{k0} - \sum_{i=1}^{m} z_i^{k*} \right) = 0 \quad \forall k \;. \tag{3.4)'$$

Comparing these conditions to those of (4.1) to (4.4), we can easily find that

$$e_i^{**} = e_i^* \,;\; \eta_{ij}^{**} = \lambda_{ij}^* \,;\; z_i^{k**} = z_i^{k*} \,;\; \pi_k^{**} = \pi_k^* \,.$$

(End of proof.)

It should be noted that the first condition (1) in Theorem 4.1 (the rank condition) identifies a technological requirement for the system. The second condition (2) (condition for initial allocation of emission licenses) describes the compatibility requirement between the emission and pollution license systems. Finally, the third condition (3) (need for private creation of emission licenses) addresses a legal and administrative requirement.

An interpretation of rank condition (1) is the following:

Suppose that m = 1 and n = 10. This is a case in which a single emission source affects ten pollution points. In this case, a change in the emission level at the source causes changes in ten pollution levels. Since the changes at pollution points are perfectly correlated to each other, no one can control these ten pollution levels individually. When the targets (or upper limits) of the pollution levels are set for these ten pollution points, the only way for these limits to be satisfied is to reduce the emission level at the single source, so that the most stringent pollution level among the ten pollution points satisfies its upper limit. This will result in an over-reduction of pollution levels at the other nine pollution points.

To make a contrast, suppose that m = 100 and n = 10. In this case, it will be easy to create a combination of emission reductions at these emission points that construct the set of ten pollution levels that are equivalent to the array of targets. Notice, however, that if these one hundred emission sources happen to have similar emission technology, the combination of these sources reduces to a hypothetical single emission source. Thus, to attain the pollution targets at all ten pollution points simultaneously, there must exist at least ten emission sources that are independent of each other. In mathematical terminology, we say that the matrix H must have at least ten linearly independent rows. That is the rank condition of (1).

5. The Control of GHG Accumulation

Montgomery's idea of introducing the "matrix of unit diffusion coefficients" originally assumed regional environmental problems in which a pollutant emitted at source point i affects the environmental quality of pollution point j. In his framework, the regulator's role was assumed to be to attain the targets at pollution points. Typical problems include air pollution, water quality, etc.

When we think of the problem of climate change, Montgomery's framework may of course be applicable to the issue. However, in this case, we have only a single pollution point, which is the globe. The matrix of unit diffusion coefficients may be reduced to the following:

$$H = [1 \quad 1 \quad \cdots \quad 1]^T \tag{5.1}$$

For regulations coping with global warming, we can introduce an international regulatory authority that establishes emission (or pollution) targets, and allocates emission (or pollution) licenses to emitting entities (basically countries). One of the current international institutions, such as the United Nations' Framework Convention on Climate Change (UNFCCC) Secretariat, may undertake this role. The regulator designated in such a way may want to take the matrix of (5.1) for granted, and to apply the Montgomery frame to the problem.

Insofar as we address the climate change problem in this way using Montgomery's model, there seems no room for any further interesting discussion: The problem we face is a simplified version of that Montgomery once faced. Rather, Montgomery's framework seems unnecessarily complicated. The current problem in the real world may be simpler than before. In short, as a theory of environmental policy issues, his view of environmental problems seems obsolete in the context of climate change.

However, if we look at the problem from a different perspective, Montgomery's model may provide a different insight. To see this, let us examine the nature of the problem once again, as follows.

Global warming is basically caused by the accumulation of greenhouse gases (GHGs) in the atmosphere. Thus, the most important thing to do should be to control the level of GHG accumulation, rather than the emission itself. In other words, to cope with climate change, we should focus on control of the GHG stock rather than that of the GHG flow each year.

To control GHG stock, we can create a market for cumulative emission permits. This idea is actually proposed by Tietenberg (2006). His Chapter 2, "The Conceptual Framework," discusses alternative permit systems, including

(i) Uniformly Mixed Assimilative Pollutants,

(ii) Nonuniformly Mixed Assimilative Pollutants, and

(iii) Uniformly Mixed Accumulative Pollutants (Cumulative Emission Permit Systems).

Analyzing the third, he shows that the prices of cumulative emission permits are to grow at the rate of the interest rate.[4] His discussion, however, does not make it clear how the cumulative emission permit system differs from typical cap-and-trade systems, in which the emission flow is subject to regulation.

In the remainder of this section, we will examine the relationship between stock and flow systems. To do so, we begin by modifying Montgomery's matrix of unit diffusion coefficients: We introduce a time horizon. Let us define it as follows:

t = 1 to T: Years

$e(t)$: GHG emission (flow) at year t

$\tau(t)$ ($\geq$1): The lifetime of the GHG that was emitted into the atmosphere at year t

$E_t(s)$: The amount of remaining GHG, at year s, out of $e(t)$ after being emitted to the atmosphere at year t

We assume that, once emitted into the atmosphere, GHG declines at the rate of δ (<1) throughout its lifetime $\tau(t)$, and it completely disappears after its lifetime. More specifically, we assume the following:

[4] This is basically Hotelling's well known rule, although Tietenberg does not explicitly mention it.

$E_t(s) = \delta^{s-t} e(t)$ for $t \le s \le \tau(t) + t - 1$, and

$E_t(s) = 0$ otherwise.

By definition, total GHG remaining in the atmosphere at year s is written as follows:

$$\sum_{t=1}^{s} E_t(s).$$

This implies that the coefficients with which GHG emissions at year t (= 1 to T) contribute to the total amount of GHG remaining in the atmosphere at year s are described as follows:

$h_{ts} = \delta^{s-t}$ for $t \le s \le \tau(t) + t - 1$, and

$h_{ts} = 0$ otherwise,

or, in a matrix form,

$$H \equiv \begin{bmatrix} 1 & \delta & \delta^2 & \cdots & \cdots & \delta^{\tau(1)-1} & 0 & 0 & 0 \\ 0 & 1 & \delta & \delta^2 & \cdots & \cdots & \delta^{\tau(2)-1} & 0 & 0 \\ 0 & 0 & \ddots & & & & & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & & & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 & \delta & \cdots & \cdots & \delta^{\tau(T)-1} \end{bmatrix},$$

which is a T by $Max\{\tau(t) + t - 1\}$ matrix.

When we introduce "the right to accumulate GHG in the atmosphere" for each year, it is a form of "stock" permit. On the other hand, when we introduce "the right to emit GHG" for each year, this is a form of "flow" permit, which is, of course, the typical emissions trading regime. One question is whether flow permit systems can replicate stock permit systems. Theorem 4.1 may help to answer this question.

Notice that the rank of matrix H defined as above is T: that is, Rank (H) = T. This means that condition (1) in Theorem 4.1 may not be satisfied automatically because $T \le Max\{\tau(t) + t - 1\}$. The following theorem states this point.

Theorem 5.1

Flow permit systems that allow for the right to emit for each year can always replicate any social planner's optimal control of pollutant accumulation only if $T = Max\{\tau(t) + t - 1\}$ *holds.*

Proof

Note that Rank (H) = T and $T \le Max\{\tau(t)+t-1\}$. Condition (1) in Theorem 4.1 is not satisfied when $T < Max\{\tau(t)+t-1\}$. Thus, $T = Max\{\tau(t)+t-1\}$ is necessary.
(End of proof.)

The above theorem leads to another theorem.

Theorem 5.2

Suppose that the lifetime τ(t) is a strictly decreasing function of t, and that there exists k such that τ(k) = 1 holds. That is,

$$\tau(t)-1 \ge \tau(t+1) \quad \forall t \le k-1, \text{ and}$$
$$\tau(t)=1 \qquad\qquad \forall t \ge k \ .$$

Then flow permit systems that allow for the right to emit for each year can always replicate any social planner's optimal control of pollutant accumulation only if $T = \tau(1)$ *holds.*

Proof

The condition $T = Max\{\tau(t)+t-1\}$ for Theorem 5.1 is equivalent to the following:

$$\tau(t)+t-1 \le T \quad \forall t .$$

By assumption,

$$\tau(1) \ge \ \cdots \ \ge \tau(t)+t-1 \ge \tau(t+1)+(t+1)-1 \ge \ \cdots \ \ge \tau(k)+k-1 = k \ .$$

Thus $Max\{\tau(t)+t-1\} = \tau(1)$. This yields the result.
(End of proof.)

Theorem 5.2 provides important implications for climate policy. The global warming phenomenon and its effects are due to GHG accumulation (or density) in the atmosphere. Coping with the effects is the best way to control GHG accumulation (or density) directly. However, controlling accumulation (or density) may not fit well into marketable permit regimes. Thus we need to resort to flow control, which leads to typical emission permit regimes.

Needless to say, these regimes must cover at least the time horizon within which emissions are observed. As long as GHG emission is indispensable to the economy, we need to maintain the permit system to extend coverage.

Theorem 5.2, however, indicates that, even if we take a long regulation time horizon, this is not enough. In addition to coverage of the time horizon, the lifetime of each year's GHG emission must decline as time goes on, and it must finally converge to unity.

If the decline of GHG lifetime occurs exogenously, it should be good for us. However, if it does not happen in such a way, we should make it happen. In other words, some form of GHG control technology is necessary, together with GHG emissions trading policy. For instance, we should utilize technologies that can capture carbon in the atmosphere, such as forestry. If it is difficult to capture carbon that has already been emitted into the atmosphere, we should accelerate those technologies that help to capture carbon at its sources. Such technology includes CO_2 Capture and Storage (CCS). Without these GHG control technologies, emissions trading will not contribute to a fundamental solution to the challenge of climate change.

6. Impacts, Adaption, and Vulnerability

Assessments of climate change can be broken down into the following three steps.

(1) To explore the scientific basis of greenhouse effects

(2) To confirm the impact of greenhouse effects on human–natural environment interactions, and to verify the possibility of adaptation

(3) To assess GHG mitigation policies

In fact, these three steps correspond to the IPCC's working groups. Working Groups 1, 2, and 3 engage in assessments of "the physical science basis," "impacts, adaptation and vulnerability," and "mitigation of climate change," respectively.

The Fourth Assessment Report (AR4) of IPCC Working Group 2 (IPCC: 2007) shows how impacts and adaptation should be treated in the climate change debate. The assessment is two-fold: in Section B, the report highlights the aspect of systems and sectors; in Section C, it focuses on regions.

The main idea of Section B is that future impacts and adaptation can be broken down into systems and sectors, as follows:

(1) Freshwater resources and their management

(2) Ecosystems, their properties, goods and services

(3) Food, fiber, and forest products

(4) Coastal systems and low-lying areas
(5) Industry, settlement and society
(6) Human health

Similarly, the main idea of Section C is that future impacts and adaptation are also broken down by regions as follows:
(1) Africa
(2) Asia
(3) Australia and New Zealand
(4) Europe
(5) Latin America
(6) North America
(7) Polar regions
(8) Small islands

Given these structures, we can again apply Montgomery's framework of the diffusion model to the problem. More specifically, for Sections B and C of the IPCC Working Group 2 AR4, the matrices of unit diffusion coefficients can be constructed, such that columns represent the systems and sectors listed above, and the regions listed above, respectively. In these cases, $n = 6$ for systems and sectors, and $n = 8$ for regions.

The question, then, is whether controlling emissions through emission permit systems may contribute to the control of these targets. Theorem 4.1 may provide an answer: In the Kyoto Protocol, emission sources are defined as the "Annex B countries,"[5] the number of which is 39 (including the US). That is $m = 39$. This means that $m > n$ for both of the cases above, and thus, Rank (H) = n holds. From the theorem, emissions trading systems similar to those defined by the Kyoto Protocol (i.e., the Kyoto mechanisms) would help attain optimal control of these targets.

Again, we are sure that trading regimes similar to the Kyoto mechanisms would work in theory. However, the real world may be different. The countries currently participating in the frame of the Kyoto mechanisms are mainly divided into three groups, excluding the US: EU countries, economies in transition (EITs), and Japan. EU countries are allowed to use the so-called "EU bubble," which makes them one entity. Although there are many countries in the EIT group, most of them have a small economy, with the exception of Russia and Ukraine. This

[5] Those countries that are obliged to reduce their own emissions are listed in Annex B in the Protocol.

situation indicates that *m* is said to be around 4, rather than 39, and thus does not satisfy the necessary condition of the theorem.

The above observation leads us to the following bottom line: If we create a similar emissions trading regime in the post-Kyoto debates, and if, at the same time, we wish to pursue the more fundamental goals of environmental protection, such as adaptation by sectors or regions, the regime may not work.

7. Conclusion

This study reconsidered the relations between two markets in licenses, *the market in licenses to pollute* and *the market in emission licenses* using the same framework as Montgomery (1972). The emphasis is upon whether these two markets are compatible, and if so, in what conditions. These questions have never been comprehensively answered by Montgomery's original work. One fundamental theorem obtained, Theorem 4.1, clarified the necessary and sufficient conditions in which *the market in emission licenses* can achieve any environmental goals at multiple locations.

Although the theorem seems abstract, it does offer important implications for future climate change policy debates, such as the post-Kyoto debates. When applied to climate policies that are intended to pursue GHG concentration targets, it emphasizes the importance of technology policy: thus current emissions market regimes, such as EU-ETS and the Kyoto mechanisms, may not be sufficient for the purpose of climate change mitigation, and some additional technology policies are needed. These notions are stated as Theorems 5.1 and 5.2. When the fundamental theorem is applied to the issue of impacts and adaptation, it provides similar indications. We may need a more comprehensive framework of emissions trading in the post-Kyoto debates.

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